Calculation of the Effect of Rectifier Transformer Impedance on Rectification Circuit Characteristics

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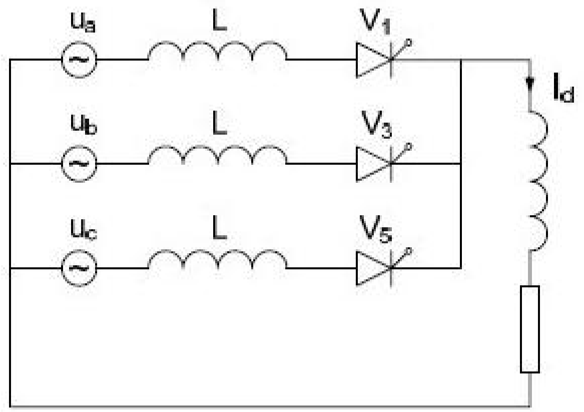
1. Calculation of Commutation Overlap Angle

As for SCR rectification circuit of m-impulse, if there is only $\frac{m}{3}$ SCR break-over at any moment, the phase difference between two phase power supply is always $\frac{m}{3}\times\frac{2\pi}{m}=\frac{2\pi}{3}$. Three-phase half-wave rectification circuit is the base unit of all three-phase rectification circuit. Hence, we take it as a base to deduce.



In the Fig.1, the letter "L" represents the leakage inductance of each phase that is converted to secondary side of power transformer. Assume that the load current is smooth direct current I_d:

$$u_a=\sqrt{2}U_{2p}\,sin(\omega t),\quad u_b=\sqrt{2}U_{2p}\,sin(\omega t-\frac{2\pi}{3}),\quad u_c=\sqrt{2}U_{2p}\,sin(\omega t+\frac{2\pi}{3})$$

The trigger angle is α; when ωt= $\frac{5\pi}{6}$ +α, commutation starts; SCR V_1 is off; the passing current reduces; the SCR V_5 begins to be on; the passing current rises from zero. The commutation is over at ωt= $\frac{5\pi}{6}$ +α+Y; the current passing through V_1 reduces to zero and cut off; the current passing through V_5 rises to I_d and the commutation overlap angle is Y. During the commutation, the load voltage is $\frac{1}{2}(u_a+u_b)$. As for the leakage inductance represented by the letter “L”:

$$L\frac{di}{dt}=\frac{1}{2}(u_a-u_b)=\frac{\sqrt{6}}{2}U_{2p}\sin(\omega t+\frac{\pi}{6}) \qquad (1)$$

$$\omega L di=\frac{\sqrt{6}}{2}U_{2p}\sin(\omega t+\frac{\pi}{6})d(\omega t) \qquad (2)$$

Perform integral to formula (2) at the section ($\frac{5\pi}{6}$ +α, $\frac{5\pi}{6}$ +α+Y):

$$\int_{I_d}^{0}\omega L di=\int_{\frac{5\pi}{6}+\alpha}^{\frac{5\pi}{6}+\alpha+\gamma}\frac{\sqrt{6}}{2}U_{2p}\sin(\omega t+\frac{\pi}{6})d(\omega t) \qquad (3)$$

$$\cos\alpha-\cos(\alpha+\gamma)=\frac{2\omega LI_d}{\sqrt{6}U_{2p}}=\frac{\sqrt{2}\omega LI_d}{U_{2l}} \qquad (4)$$

Usually, it is impossible to get the leakage reactance of each phase. The formula (4) is impractical. Now that the impedance of rectifier transformer - $u_k\%$ is high and leakage reactance is far more than winding resistance, the leakage inductive reactance is leakage reactance. According to the definition of short-circuit impedance, the following formula is obtained:

$$\omega L=\frac{u_{2p}}{I_2}\times u_k\%=\frac{U_{2l}}{\sqrt{3}I_2}\times u_k\% \qquad (5)$$

Substitute (5) into (4)

$$\cos\alpha-\cos(\alpha+\gamma)=\frac{\sqrt{6}I_d}{3I_2}\times u_k\% \qquad (6)$$

I_2 represents rated current of transformer secondary side. Once the transformer is produced, its impedance will not change if its structure and shape remain the same. Hence, based on the formula (6), it is found that commutation overlap angle represented by Y is only related to trigger angle and load current - I_d. The formula is applicable to three-phase half-wave rectification circuit, three-phase full-bridge rectification circuit, single-phase full-bridge rectification circuit and rectification circuit composed of these several circuits.

2. Calculation of Output Voltage

The output waveform of m-impulse rectification circuit is shown in Fig.2. The output voltage waveform between $\frac{\pi}{2}+\frac{\pi}{m}+\alpha$ and $\frac{\pi}{2}+\frac{3\pi}{m}+\alpha$ is selected to calculate the average value of output voltage. The output waveform is composed of two sections of sine waves.

Fig.2 Output waveform of m-impulse rectification circuit

$$U_d=\frac{1}{\frac{2\pi}{m}}\left[\int_{\frac{\pi}{2}+\frac{\pi}{m}+\alpha}^{\frac{\pi}{2}+\frac{\pi}{m}+\alpha+\gamma}\sqrt{2}U\cos\left(\frac{\pi}{2m}\right)\sin\left(\omega t-\frac{\pi}{m}\right)d(\omega t)\right.$$

$$\left.+\int_{\frac{\pi}{2}+\frac{\pi}{m}+\alpha+\gamma}^{\frac{\pi}{2}+\frac{3\pi}{m}+\alpha}\sqrt{2}U\sin\left(\omega t-\frac{2\pi}{m}\right)d(\omega t)\right]$$

$$=\frac{\sqrt{2}mU}{\pi}\sin\left(\frac{\pi}{m}\right)\cos\left(\frac{\gamma}{2}\right)\cos\left(\alpha+\frac{\gamma}{2}\right) \qquad (7)$$

Substitute m=3 into the formula (7) and the formula of output voltage of three-phase full-bridge rectification circuit is listed below:

$$U_d=\frac{3\sqrt{2}}{\pi}U_{2l}\cos\frac{\gamma}{2}\cos\left(\alpha+\frac{\gamma}{2}\right) \qquad (9)$$

Where:

U_{2l}: effective value of line voltage

3. Calculation of Input Power Coefficient

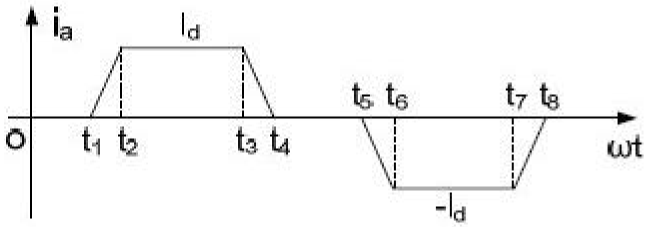
Take three-phase full-bridge rectification circuit as an example. The rectifier transformer output current is smooth direct current - I_d and the commutation overlap angle is represented by the symbol Y. Then the waveform of phase A current is shown in Fig.3.

Analyze the waveform shown in Fig.3 and we can get the following：

$$I=\sqrt{\frac{1}{3\pi}(2\pi-\gamma)}I_d \qquad (10)$$

$$i_{a1}=\frac{4\sqrt{3}}{\gamma\pi}sin(\frac{\gamma}{2})I_d\ sin[\omega t-(\alpha+\frac{\gamma}{2})] \qquad (11)$$

$$\phi=\alpha+\frac{\gamma}{2} \qquad (12)$$



$t_1:\alpha+\frac{\pi}{6}$; $t_2:\alpha+\gamma+\frac{\pi}{6}$; $t_3:\alpha+\frac{5\pi}{6}$;

$t_4:\alpha+\gamma+\frac{5\pi}{6}$; $t_5:\alpha+\frac{7\pi}{6}$; $t_6:\alpha+\gamma+\frac{7\pi}{6}$;

$t_7:\alpha+\frac{11\pi}{6}$; $t_8:\alpha+\gamma+\frac{11\pi}{6}$

Fig.3 Current waveform of secondary side phase A

Distortion factor:

$$\xi=\frac{I_1}{I}=\frac{12}{\gamma}sin(\frac{\gamma}{2})\sqrt{\frac{1}{2\pi(2\pi-\gamma)}} \qquad (13)$$

Total power coefficient:

$$\begin{aligned}cos\ \varphi&=\xi\ cos\ \phi\\&=\frac{12}{\gamma}sin(\frac{\gamma}{2})cos(\alpha+\frac{\gamma}{2})\sqrt{\frac{1}{2\pi(2\pi-\gamma)}}\end{aligned} \qquad (14)$$