Theoretical Calculation and Test Research on Thermal Time Constant of Single-core Cables

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Abstract: Owing to the existence of cable heat capacity, the temperature of cable alters gradually as the time passes when applying the step current. The temperature achieves the steady state after a certain time. The speed of temperature change of cable conductor is reflected by thermal time constant. Hence, the single-cable cable is taken as an object of study and thermal time constant of cable is introduced. The transient thermal circuit models of cable body and ambient media are established and simplified; thermal time constant of single-core cables in air and buried in ground is calculated; the temperature rise test of single-core cable is conducted on site at step current. The actual thermal time constant of cables is obtained by curve fitting of real transient process of conductor temperature, which proves the correctness of theoretical calculation. The thermal time constant of cable can be used to estimate the transient response of conductor temperature and to determine the needed time of achieving highest allowable conductor temperature of cables when step current is applied. That provides theoretical support for cable condition monitoring and early fault warning during cable operation.

Key words: Conductor temperature, thermal time constant, single-core cable, transient thermal circuit model, calculation, test, temperature rise time

Introduction

The conductor temperature of cables in operation is the decisive factor in affecting the lifespan of insulation material. The status of cables in operation can be monitored via the temperature of cable conductor. However, due to technical restrictions, it is difficult to measure the temperature of operating cable conductor accurately. Hence, it is common practice to use theoretical calculation to obtain the conductor temperature.

When the single-core cable works continuously at rated carrying capacity, its components including conductor, insulation and metal sheath will produce losses, forming the steady-state temperature field. The temperature of conductor can be calculated via steady-state thermal circuit and its calculation method is given by IEC60287 standard.

But in actual operation, when a sudden change occurs to the cable load current, the temperature alters gradually owing to the existence of
internal heat capacity and then becomes stable after a certain time. Before achieving the balance, the cable is in the non-stable state temperature field. The thermal time constant is used to reflect the speed of temperature change of cable conductor. Generally speaking, the cable system can be equivalent to thermal resistance and heat capacity in series and the response is exponential curve. Based on the thermal time constant, we can get the formula of conductor temperature response and then calculate the temperature of cable.

1. Transient Response and Thermal Time Constant of Typical First-order Circuit

The physical quantities in the heat flow field and thermal circuit correspond with those of electric circuit. As a result, the knowledge of current field and circuit can be used to analyze the thermal performance of cable. The transient thermal circuit of cable is similar to \( R_c C_e \) series circuit. When the current \( I \) passes through the cable conductor, the heat energy transfers to all directions from the center of conductor. Its thermal process can be equivalent to thermal resistance and heat capacity of all cable layers. The cable system including cable body and ambient media can be equivalent to one thermal resistance \( R \) and one heat capacity in series shown in Fig.1.

![Diagram](image.png)

Fig.1 F First-order RC transient thermal circuit

\( Q_c \) -- heat flow of conductor is represented by and its value is \( I^2 R_e \)

\( \theta_c \) -- unknown conductor temperature

\( \theta_0 \) -- known environment temperature

The thermal balance equation of thermal circuit is established:

\[
Q_c = C \frac{d\theta_c}{dt} + \frac{\theta_c - \theta_0}{R} \tag{1}
\]

\[
\theta_c = Q_e R + \theta_0 + Ae^{-t/RC} \tag{2}
\]

According to initial condition: \( t=0, \theta_c = \theta_0 \), particular solution of equation is listed below:

\[
\theta_c = \theta_0 + Q_e R (1 - e^{-t/RC}) \tag{3}
\]

RC is called thermal time constant and represented by the symbol \( \tau \). The \( \theta_c \) formula is drawn into curve, which is shown in Fig.2. The thermal time constant reflects the speed in which temperature rise of cable varies and is one of characteristic parameters of cable system.
2. Establishment of Transient Thermal Circuit Model of Cable and Ambient Media

The single-core cable is composed of conductor, insulation, metal coating and external sheath. The heat transfer characteristics vary according to the type of material. In the transient thermal circuit, it is necessary to consider thermal resistance of insulation and external sheath, and each layer’s heat capacity. Besides, it also includes thermal resistance and capacity of ambient media. Therefore, as to actual cable thermal circuit, the thermal time constant cannot be directly calculated as the circuit shown in Fig.1. However, the transient thermal circuit model of cable with distribution parameters can be built, and the same goes for transient thermal ambient media. The distribution parameters are combined into lump parameters to obtain the thermal time constant of cable.

2.1 Transient thermal circuit model of cable body

In the engineering, it is common standard to use the second-branch network to stimulate the transient thermal circuit of cable body. The allocation proportion factors p and p’ divide heat capacity of insulation-Ci and external sheath Cj to their adjacent structure; the effect of thermal current of metal coating is converted to corresponding thermal resistance and heat capacity parameters by means of the coefficient q. The simplified equivalent thermal circuit is shown in Fig.3.

Fig.3 Equivalent thermal circuit of cable body
θc -- temperature of conductor
θs -- temperature of metal coating
θj -- temperature of surface

2.2 Transient thermal circuit model of ambient media

The existence of thermal resistance and heat capacity of ambient media will have an influence on thermal time constant of cable system. As to direct-buried cable, it is tough to establish its transient thermal circuit due to the complexity of soil structure and heat characteristics.

The heat capacity should be considered in the transient thermal circuit. To reflect the temperature rise of cable well, the soil thermal circuit of four times of buried depth should be further categorized. The soil can be categorized into four areas and the
transient thermal circuit of soil is shown in Fig.4.

Fig.4 Transient thermal circuit of soil around cables

The method of categorizing four areas of the soil is given as follows: the internal diameter of the first area is equal to external diameter of cable De while the external diameter of the fourth area \( D_4 = 4L \). Then divide the section \( \frac{\pi}{4} (D_4^2 - D_1^2) \) according to the volume ratio 1:4:16:64 into four areas. The external diameter of the first area to third area meets the following requirements:

\[
A = \frac{\pi}{4} (D_4^2 - D_1^2)
\]

\[
D_1 = \sqrt{\frac{4}{\pi} A + D_1^2}
\]

\[
D_2 = \sqrt{\frac{16}{\pi} A + D_1^2}
\]

\[
D_3 = \sqrt{\frac{64}{\pi} A + D_2^2}
\]

If the thermal resistance and heat capacity coefficients of soil are known, thermal resistance \( R_{11}, R_{44} \) and heat capacity \( C_{41}, C_{44} \) of corresponding area can be calculated. The simplified thermal circuit of soil is shown in Fig.5.

Fig.5 Equivalent transient thermal circuit of soil

\[
P_{4,n} = \frac{1}{21n(D_n/D_{n-1})} \cdot \frac{1}{(D_n/D_{n-1})^2 - 1}
\]

\( n = 1-4; \ D_0 = D_e \).

Formulas of thermal resistance and heat capacity are presented below:

\[
R_{D,n+1} = \frac{q_s}{R_{4,n+1}}
\]

\[
C_{D,n} = \frac{(1-p_{4,n})C_{4,n} + p_{4,n+1}C_{4,n+1}}{q_s}
\]

As to the cable laid in the air, the heat capacity of air can be neglected. And the only thing that needs to be considered is the effect of thermal resistance on thermal time constant.

2.3 Equivalent change of cable transient thermal circuit

Combine equivalent thermal circuit of cable body shown in Fig.3 with equivalent thermal circuit of soil shown in Fig.5. A new circuit is shown in Fig.6.

Fig.6 Equivalent change of cable transient thermal circuit
Fig.6 Equivalent thermal circuit of cable buried in soil

In the Fig.6, $C'_{D0} = C_B + C_{D0}$. In order to calculate the thermal time constant, it is necessary to convert thermal circuit into the one shown in Fig.1. The thermal resistance and heat capacity are changed to lump parameters.

$$R = R_A + R_B + R_{D1} + R_{D2} + R_{D3} + R_{D4}$$

$$C = C_A + \left( \frac{R_B}{R} + \frac{R_{D1}}{R} + \frac{R_{D2}}{R} + \frac{R_{D3}}{R} + \frac{R_{D4}}{R} \right)^2 C_B$$

$$C_D + \left( \frac{R_{D1}}{R} + \frac{R_{D2}}{R} + \frac{R_{D3}}{R} + \frac{R_{D4}}{R} \right)^2 C_{D0}$$

As for the cable in air, the only thing that you need to do is to convert $R_B$ in the Fig.3 into $R_B + R_4$. The heat capacity of air is neglected. Then the value of $C$ is obtained. The time constant of transient thermal circuit of cable system $\tau = RC$.

3. Theoretical Calculation of Thermal Time Constant

3.1 Calculation of thermal time constant of single-core direct-buried cable

The theoretical calculation of thermal time constant of one single-core direct-buried cable is calculated in this paper. The cable model is YJLW0364/110 1x630. Standard parameters of thermal circuit shown in Tab.1 are obtained through careful investigation of cable dimension and thermal resistivity. The soil thermal resistivity is 0.6K·m/W.

Tab.1 Standard parameters for calculation on thermal time constant of direct-buried cable

<table>
<thead>
<tr>
<th>Dimension/mm</th>
<th>Thermal resistance</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d_c$</td>
<td>$D_1$</td>
</tr>
<tr>
<td>30.3</td>
<td>74</td>
</tr>
</tbody>
</table>

The formula of heat capacity and volume ratio coefficients can be found in the IEC standards. The heat capacity of each layer can be calculated out based on parameters listed in Tab.1. As to single-core cable, its eddy-current loss can be neglected. Hence, the thermal current of metal sheath $q_a \approx 0$ and $q_q \approx 1$. The thermal resistance and heat capacity of equivalent thermal circuit of cable body can be calculated.

As to the thermal circuit of soil, $D_4 = 4L = 4.0m$, by means of the formula (4), $D_1 = 0.4451m$, $D_2 = 0.975m$, $D_3 = 1.9901m$. The thermal coefficient of soil is 0.6K·m/W; the coefficient of volume ratio heat capacity is 2.52J/cm³K; the thermal resistance and heat capacity of each layer can be calculated. Furthermore, based on the formulas (5) and (6), thermal resistance and heat capacity of equivalent thermal circuit are calculated:

$$C'_{D0} = 1.0 \times 105 J/(K\cdot m)$$

$$C_{D1} = 8.3 \times 105 J/(K\cdot m)$$

$$C_{D2} = 3.2 \times 106 J/(K\cdot m)$$

$$C_{D3} = 1.3 \times 107 J/(K\cdot m)$$
The temperature at the steady state rises by 12.4 °C; the temperature only goes up by 1.2 °C around the cable (1-4m). From the above calculation, it is found that the heat capacity of soil is more than that of cable body; the heat capacity of third layer is much higher than total heat capacity of cable body. To simplify the calculation of thermal time constant, only former layers of soil are considered in this paper.

According to the formula (7), it can be calculated that \( R = 1.08 \text{Km/W}, \) \( Q = 13682/(\text{Km}) \). The theoretical value of thermal time constant of single-core cable \( \tau = RC = 14777\text{s} = 4.1\text{h} \).

3.2 Calculation of thermal time constant of single-core cable in air

The theoretical calculation of thermal time constant of single-core cable buried in air was made. The model of cable YJLW03 290/500 1×2500 and the dimension of cable is listed in Tab.2.

<table>
<thead>
<tr>
<th>Item</th>
<th>dc</th>
<th>Di</th>
<th>Ds</th>
<th>De</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diameter/m</td>
<td>61.2</td>
<td>138.1</td>
<td>155.8</td>
<td>175.8</td>
</tr>
<tr>
<td>mm</td>
<td></td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>

According to IEC standards, thermal resistance and heat capacity of cable equivalent thermal circuit and air outside thermal resistance can be calculated out. According to the formula (7), we can calculate that \( R = 1.04 \text{Km/W}, \) \( C = 20251/(\text{Km}) \). The theoretical value of single 2500mm² cable's thermal time constant is calculated at 5.9h \( (\tau = RC = 21062\text{s}) \).

4. Test of Single Cable Thermal Time Constant

To verify the theoretical evaluation of thermal time constant, temperature rise tests of single direct-buried cable and cable in air were respectively conducted.

4.1 Temperature rise test of single direct-buried cable

The single 630mm² cable is directly buried underground, which is 1m from the ground. Add 1250A step load and continuously monitor the temperature of cable conductor for 40 hours of increasing the current. Discrete points are shown in Fig.7.

![Fig.7 Test results and exponential fitting curve of conductor temperature rise of single-core cable buried underground](image)

Based on test results of conductor temperature, the exponential curve is fitted shown in Fig.7. Its formula is listed below:

\[
\theta_c = 13 + 60(1-e^{-t/4})
\]
The actual thermal time constant of 630mm² single core direct-buried cable is 4h according to the formula (8). The time constant via theoretical calculation is 4.1h, which proves that it is correct just to consider the two former soil layers when calculating the thermal time constant of single direct-buried cable.

When several direct-buried cables run in parallel, the thermal effect among cables caused by less than 1m cable distance decreases the thermal time constant of cable system and accelerates its temperature rise. That can be expected. In the temperature rise test of single-circuit three-phase direct-buried cable, let’s take middle-phase conductor for example. The 800A constant current passes through the cable until the temperature of conductor remains at 62°C. After 94h, apply 1200A step current. For the data of temperature rise test, see discrete points shown in Fig.8. The exponential curve is fitted and its formula is given below:

\[ \theta_c = 62 + 31(1-e^{-\frac{(t-94)}{3}}), t\geq 94h \quad (9) \]

Fig.8 Test results and exponential fitting curve of middle conductor temperature rise of three-phase cable buried underground

Based on the formula (9), the actual thermal time constant of three-phase 630mm² cable system is 3h, which is less than that of single direct-buried single-core system.

4.2 Temperature rise test of single-core cable in air

The 2500mm² single-core cable in air coupled with step current is 2350A. The discrete points shown in Fig.9 illustrate test results of conductor temperature.

Fig.9 Test results and exponential fitting curve of middle conductor temperature rise of single-core cable in air

Based on test results of conductor temperature, the exponential curve is fitted shown in Fig.9. Its formula is listed below:

\[ \theta_c = 33 + 61(1-e^{-t/6.2}) \quad (10) \]

The thermal time constant of 2500mm² single-core cable in air is 6.2h while the value is 5.9h via theoretical calculation. That proves that the theoretical calculation of thermal time constant of cable in air is
5. Conclusions

a) The thermal time constant of cable is determined together by thermal resistance and heat capacity of cable thermal circuit and ambient media. The thermal resistance and heat capacity parameters of cable body can be calculated according to the cable structure and parameters. The thermal resistance and heat capacity of ambient media is very complicated. As to the cables buried in the soil, relevant simplification and layer division should be performed. In terms of the cable in air, the heat capacity is generally neglected. Only the effect of air thermal resistance on the time constant is considered.

b) According to the thermal time constant of cable system, the response of cable conductor temperature under the influence of emergency load can be estimated. Then the temperature of conductor at some moment is estimated. The calculation method based on optical monitoring surface temperature is more accurate.

c) The estimation of thermal time constant of cable system is helpful for studying the transient heat issue under short-time over-load or short-circuit breakdown. It can also be used for determine how long it takes to reach highest allowable conductor temperature at step current, which provides theoretical support for early cable fault warning.